Continuous measurements in a composite quantum system and possible exchange of information between its parts.

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We study an influence of the continuous measurement in a composite quantum system C on the evolution of the states of its parts. It is shown that the character of the evolution (decoherence or recoherence) depends on the type of the measured quantity and on the initial state of the system. A number of conditions under which the states of the subsystems of C decohere during the measuring process are established. We propose a model of the composite system and specify the observable the measurement of which may result in the recoherence of the state of one of the subsystems of C. In the framework of this model we find the optimal regime for the exchange of information between the parts of C during the measurement. The main characteristics of such a process are computed. We propose a scheme of detection of the recoherence under the measurement in a concrete physical experiment.

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The main goal of the paper is to point out a possibility of the recoherence of the state of one of the parts of a composite quantum system under the measurement of physical quantities of a certain type. Another words, the decoherence of the state of the composite system, caused by the interaction with the environment, can be accompanied, under certain conditions, with the recoherence, or purification, of a mixed state of one of its parts.

The paper is organized as follows. First, we review shortly some background knowledge of the theory of open quantum systems (OQS) and of the theory of continuous quantum measurements (CQM) used in the paper. The main object under study, the composite system C that consists of non-interacting parts, is introduced, and the evolution of the states of the subsystems of C during the continuous measurement is considered. A number of conditions under which the measurement of C results in the decoherence of states of all its parts are formulated. Then, we consider a more interesting situation, when the measurement in the composite system may result in the recoherence of the state of one of the subsystems. In the case when such a recoherence takes place, one can say about the exchange of information between the subsystems during the measuring process. On the example of a simple model of the composite system we study the influence of the initial states of the parts of C on the process of the exchange of information between the parts under the measurement of the observable that has only two eigenvalues and commutes with the Hamiltonian of C. We find the optimal regime of the information exchange and compute the main characteristics of such a process: ΔI_{max}, ΔS and η, where ΔI_{max} is the maximum amount of information, received by one subsystem (receiver), ΔS, the increment of the entropy of the other subsystem (sender) in the same process, and $\eta = \Delta I_{max}/\Delta S$, the efficiency coefficient for the information exchange under the measurement. We also consider the special regime of the information exchange under which the energies of the subsystems do not change under the measurement. In the concluding part of the paper we propose the scheme for the observation of the recoherence in a concrete physical experiment.

Let us go to the details.

During last years the point of view, formulated most precisely by Zurek [1], becomes more and more widespread in the quantum physics community. According to it, the behavior of a quantum system becomes more classical due to its interaction with the macroscopic environment. As a rule, this interaction results in the decoherence of the state of the quantum system, i.e., the transformation of initially pure states into mixed ones. Thus, the decoherence suppresses the possibility of interference of quantum states and changes the initial picture of propagating probability waves to the usual statistical description. The idea of the decoherence is basic also for the theory of continuous quantum measurements. The important conceptual advantage of this theory is the possibility to consider the measurement as the specific process of interaction of two systems: the measured (quantum) system and the measuring (device) system. The measuring system can be as classical ones as well as mesoscopic ones, and, in the framework of the CQM theory, the description of the measurement is based on the general principles of quantum mechanics and does not involve the ideas of the wave function collapse under the measurement, etc. In the most general form the CQM theory has been formulated in the papers by Mensky [2] basing on the method of restricted Feynman path integrals (the method of quantum corridors). But for the important class of the problems, connected with the analysis of the behavior of quantum systems under nonselective measurements, another more simple method can be used. The method is based on the Lindblad equation and yields the same results. Let us remind that at nonselective measurements, in difference with selective ones, we are interesting not in the result of the measurement, but only in the influence of the measuring procedure on the state of the measured system. In general case, the

Lindblad equation [3], that describes the evolution of the density matrix of an open quantum system (OQS), has the following form:

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar}[\hat{H}, \hat{\rho}] + \frac{\gamma}{2}\left\{[\hat{R}\hat{\rho}, \hat{R}^{+}] + [\hat{R}, \hat{\rho}\hat{R}^{+}]\right\} \qquad (1)$$

(in what follows we use the systems of units, where $\hbar = 1$).

The first term in the right hand side of Eq.(1) is connected with the internal (Hamilton) dynamics of OQS, and the second term describes its evolution caused by the interaction with the environment. A concrete form of the operators $\hat{R}$ and $\hat{R}^{+}$ in Eq. (1) is determined by reduction of description of the closed system ("the system under investigation" + "the environment") with respect to the environment variables, or directly from the physical reasons. As was shown, for the first time, by Lindblad [3], Eq. (1) for $\hat{\rho}(t)$ is the only possible equation of the Markov type that satisfies the main postulates of quantum mechanics: 1) the superposition principle (linearity in $\hat{\rho}$); 2) the conservation of the total probability ($tr(\hat{\rho}) = 1$); and 3) the condition of non-negativity of $\hat{\rho}$: $\langle\psi|\hat{\rho}(t)|\psi\rangle \geq 0$ for all $t \geq 0$. Let us show with the use of Eq. (1) how the interaction of the quantum system with the environment (or the form of the operators $\hat{R}$ and $\hat{R}^{+}$) determines the character of evolution of its states, in particular, their possible decoherence or recoherence. To do this, we introduce the linear entropy $S[\hat{\rho}(t)] = tr(\hat{\rho}(t) - \hat{\rho}^2(t)) \equiv 1 - tr(\hat{\rho}^2(t))$, the quantity that characterizes a degree of "purity" (i.e. the coherence) of the quantum state with the density matrix $\hat{\rho}(t)$. One can show that the linear entropy has the properties similar to ones of the von Neumann entropy $S_{vN} = -tr(\hat{\rho}(t)\ln\hat{\rho}(t))$[4]. But, for the analysis of the evolution of $\hat{\rho}(t)$ in the framework of Eq. (1) the linear entropy serves as more convenient measure of the coherence of the state. Let us consider now the important special case of Eq. (1), when the operators $\hat{R}$ and $\hat{R}^{+}$ commute with each other. One can prove (see Appendix A) that the linear entropy of such OQS increases with time $dS[\hat{\rho}]/dt \geq 0$. This result is valid for any hermitian operator ($\hat{R} = \hat{R}^{+} = \hat{O}$). In this case the Lindblad equation (without the Hamilton term) can be written in the form:

$$\frac{d\hat{\rho}}{dt} = -\frac{\gamma}{2}[\hat{O}, [\hat{O}, \hat{\rho}]]. \qquad (2)$$

According to the CQM theory (see [2]), it is just the equation that describes the evolution of a quantum system under a continuous non-selective measurement of an observable $\hat{O}$. It is necessary to emphasize the difference of the von Neumann and the CQM theory schemes of description of the measurement. In the von Neumann approach the measurement of the observable $\hat{O}$ is an instant process that results in a sharp change (collapse) of the state of the system. In the CQM theory the same final state is reached for the finite time $\Delta t_0 \sim \gamma^{-1}$. This time is neglected in the von Neumann scheme. The law of increasing of entropy ($dS[\hat{\rho}(t)]/dt \geq 0$) reflects the irreversible character of the measurement and witnesses for the decoherence of the state of the measured object. One can note that, according to Eq. (1), for a certain type of interaction of OQS with the environment the recoherence of its state may also take place. As the simplest example, we consider a two dimensional OQS with the $|g\rangle$ and $|e\rangle$ orthonormal basis states. We choose the operator $|g\rangle\langle e|$ as the $\hat{R}$ operator, and, correspondingly, the $|e\rangle\langle g|$ operator as the $\hat{R}^{+}$ operator. Using Eq. (1) without the Hamilton term, one can show that the unique stationary state of the system is the pure state $\hat{\rho}_{st} = |g\rangle\langle g|$. This is the attractive state: $\lim_{t\to\infty}\hat{\rho}(t) = \hat{\rho}_{st} = |g\rangle\langle g|$ for any initial state $\hat{\rho}(0)$. Therefore, beginning from a certain time, the recoherence of such OQS will take place. This example can be generalized for OQS of an arbitrary finite dimension N. Moreover, for any fixed pure state of that system one can construct the corresponding interaction with the environment that provides evolution of any initial state to that pure state.

Let us now formulate the problem of interest. We consider a composite quantum system C consisting of such two parts R and S that the interaction between these parts can be neglected. Let one measures an observable $\hat{O}_C$ of the composite system. According to the foregoing statement, any state of C decoheres during the measurement. But the following question emerges: how does the states of R and S evolve under the measurement? Obviously, there are two possibilities (alternatives) A1 and A2 in such a situation.

A1. The decoherence of the state of the composite system C is accompanied with the decoherence of both its parts R and S.

A2. In spite of the decoherence of the state of C, the purification (or the recoherence) of an initial state of one part (for definiteness, R) of the composite system takes place.

In this paper we are mainly interested in the alternative A2. If this possibility is realized one can say that an information is transferred from S to R during the measurement. It is natural to call the subsystem R as the receiver and the subsystem S as the sender. Prior the study of the possibility A2 we should point out a number of conditions that exclude its realization.

One can prove (see Appendix B) that the measurement of an additive observable $\hat{O}_C = \hat{A}_R \otimes \hat{1}_S + \hat{1}_R \otimes \hat{B}_S$ results in the decoherence of the state of R and S for any initial state $\hat{\rho}_C(0)$. Another case of realization of the possibility A1 is the measurement of a multiplicative observable $\hat{O}_C = \hat{A}_R \otimes \hat{B}_S$ under condition that the initial states of the subsystems R and S are uncorrelated: $\hat{\rho}_C(0) = \hat{\rho}_R(0) \otimes \hat{\rho}_S(0)$ (see Appendix B).

Now we consider the simplest model of the composite system C and construct the observable $\hat{O}_C$ the measurement of which, under certain initial conditions, realizes the case A2. Let the Hamiltonian of C (with noninter-

acting parts R and S) has the form

$$\hat{H}_C = \hat{H}_R \otimes \hat{1}_S + \hat{1}_R \otimes \hat{H}_S, \tag{3}$$

where $\hat{H}_R$ and $\hat{H}_S$ are the Hamiltonians of the subsystems R and S.

Let us assume now that the subsystems S and R are of the same dimension: $\dim R = \dim S = N$. We also imply that the Hamiltonians $\hat{H}_S$ and $\hat{H}_R$ are unitary equivalent: $\hat{H}_S = \hat{U}\hat{H}_R\hat{U}^+$, where $\hat{U}\hat{U}^+ = \hat{U}^+\hat{U} = \hat{1}$. We define the operator $\hat{O}_C$ as

$$\hat{O}_C = (\hat{U}^+ \otimes \hat{U})\hat{T}, \tag{4}$$

where $\hat{T}$ is the hermitian operator that permutes the states of the subsystems R and S. The action of the operator $\hat{T}$ on the basis states of C is defined by the equation:

$$\hat{T}|i\rangle_R \otimes |j\rangle_S = |j\rangle_R \otimes |i\rangle_S. \tag{5}$$

Three essential properties of the operator $\hat{O}_C$ follows directly from the definitions (4) and (5): a) $\hat{O}_C = \hat{O}_C^+ = \hat{T}(\hat{U} \otimes \hat{U}^+)$; b) $\hat{O}_C^2 = \hat{1}_C$; c) $[\hat{O}_C, \hat{H}_C] = 0$.

The property a) shows that $\hat{O}_C$ is the hermitian operator and, consequently, $\hat{O}_C$ is the observable. It follows from b) that the eigenvalues of $\hat{O}_C$ are equal to 1 or -1. The property c) indicates that the total energy of the composite system C conserves under the measurement of $\hat{O}_C$ (while the energies of its parts may not conserve). Besides that, we note that the commutativity of $\hat{O}_C$ and $\hat{H}_C$ results in that the evolution of the density matrix $\hat{\rho}_C$ is totally determines by the process of the measurement of $\hat{O}_C$. Indeed, let us introduce the density matrix $\hat{W}_C$ connected with $\hat{\rho}_C$ by the unitary transformation $\hat{\rho}_C = e^{-i\hat{H}_C t}\hat{W}_C e^{i\hat{H}_C t}$. As is easily seen, the matrix $\hat{W}_C$ satisfies the same equation as the matrix $\hat{\rho}_C$ but without the Hamilton term:

$$\frac{d\hat{W}_C}{dt} = -\frac{1}{2}\left[\hat{O}_C, \left[\hat{O}_C, \hat{W}_C\right]\right]. \tag{6}$$

Eq. (6) for $\hat{W}_C$ is the master equation of the CQM theory, written in the dimensionless form. Let us introduce now the density matrices $\hat{\rho}_R$ and $\hat{\rho}_S$ that describe the states of the parts R and S of the system C and derive the equations of their evolution under the measurement of $\hat{O}_C$. By definition, $\hat{\rho}_R = tr_S(\hat{W}_C)$ and $\hat{\rho}_S = tr_R(\hat{W}_C)$. Taking the traces of the left hand side and the right hand side of Eq. (6) over the states of the subsystem S and using the properties of the operator $\hat{O}_C$ we find the equation of evolution for $\hat{\rho}_R$:

$$\frac{d\hat{\rho}_R}{dt} = \hat{U}^+\hat{\rho}_S\hat{U} - \hat{\rho}_R. \tag{7}$$

Analogously, we obtain the equation for $\hat{\rho}_S$:

$$\frac{d\hat{\rho}_S}{dt} = \hat{U}\hat{\rho}_R\hat{U}^+ - \hat{\rho}_S. \tag{8}$$

Thus, in this model, the measurement of the observable $\hat{O}_C$ (4) results in the simple picture of evolution of the states of the subsystems R and S of the composite system C. If the initial states $\hat{\rho}_S(0)$ and $\hat{\rho}_R(0)$ are specified, the system of equations (7), (8) for $\hat{\rho}_S(t)$ and $\hat{\rho}_R(t)$ allows to determine the states of the parts S and R at an arbitrary time t. For further consideration we need to know the relation between the final states $\hat{\rho}_R(\infty) = \lim_{t\to\infty} \hat{\rho}_R(t)$, $\hat{\rho}_S(\infty) = \lim_{t\to\infty} \hat{\rho}_S(t)$ and the initial states $\hat{\rho}_R(0)$, $\hat{\rho}_S(0)$. Integrating the system (7), (8) and approaching the limit as $t \to \infty$ we obtain the desired relation in the form of two equations

$$\hat{\rho}_R(\infty) = \frac{\hat{\rho}_R(0) + \hat{U}^+\hat{\rho}_S(0)\hat{U}}{2}, \tag{9}$$

$$\hat{\rho}_S(\infty) = \frac{\hat{\rho}_S(0) + \hat{U}\hat{\rho}_R(0)\hat{U}^+}{2}. \tag{10}$$

Now we have all that is needed for the study of the process of the exchange of information between the parts R and S under the measurement of the observable $\hat{O}_C$. We consider this problem in the following formulation. Let at time $t = 0$ (the starting time of the measurement) the state of the sender $\hat{\rho}_S(0)$ is known. We will find the initial state of the receiver $\tilde{\hat{\rho}}_R(0)$ that provides the maximum amount of information ΔI_R transferred from S to R during the measurement. The increment of the amount of information in the subsystem R for the time of the measurement can be written in the form:

$$\begin{aligned}\Delta I_R = -\Delta S_R = S_R\left[\hat{\rho}_R(0)\right] - S_R\left[\hat{\rho}_R(\infty)\right] \\ = tr(\hat{\rho}_R^2(\infty)) - tr(\hat{\rho}_R^2(0)).\end{aligned} \tag{11}$$

Substituting Eq. (9) into Eq. (11) we find the dependence of ΔI_R on the initial states of R and S

$$\Delta I_R = -\frac{3}{4}tr(\hat{\rho}_R^2(0)) + \frac{1}{4}tr(\hat{\rho}_S^2(0)) + \frac{1}{2}tr(\hat{\rho}_R(0)\hat{U}^+\hat{\rho}_S(0)\hat{U}). \tag{12}$$

Computing the maximum of the quadratic functional Eq. (12) with respect to $\hat{\rho}_R(0)$ under the additional restriction $tr(\hat{\rho}_R(0)) = 1$ we determine the desired density matrix $\tilde{\hat{\rho}}_R(0)$

$$\tilde{\hat{\rho}}_R(0) = \frac{1}{3}\hat{U}^+\hat{\rho}_S(0)\hat{U} + \frac{2}{3N}\hat{1}_R \tag{13}$$

and the maximum amount of information ΔI_R transferred from S to R in the optimal regime for the given initial state of the part S

$$\Delta I_R^{max}\left\{\hat{\rho}_S(0)\right\} = \frac{1}{3}\left[tr(\hat{\rho}_S^2(0)) - \frac{1}{N}\right]. \tag{14}$$

As follows from the expression (14), for any initial state of the sender (excluding the disordered one $\hat{\rho}_S(0) = \hat{1}_S/N$) one can find initial states $\hat{\rho}_R(0)$ for which the amount of information ΔI received during the measurement will be

positive. The global maximum of the amount of information transferred from S to R is reached for the pure initial state of S and it is equal to

$$\Delta I_{max} = \frac{1}{3}\left(1 - \frac{1}{N}\right) \qquad (15)$$

Another essential characteristics of the information transfer process is the increment of the entropy of the sender ΔS. Let us find this quantity for the optimal regime of the information exchange considered above.

$$\begin{aligned}\Delta S \equiv S\left[\hat{\rho}_S(\infty)\right] - S\left[\hat{\rho}_S(0)\right] \\ = tr(\hat{\rho}_S^2(0)) - tr(\hat{\rho}_S^2(\infty)) \\ = \frac{3}{4}tr(\hat{\rho}_S^2(0)) - \frac{1}{4}tr(\tilde{\hat{\rho}}_R^2(0)) - \frac{1}{2}tr(\hat{\rho}_S(0)\hat{U}\tilde{\hat{\rho}}_R(0)\hat{U}^+)\end{aligned} \qquad (16)$$

Note that we use Eq. (10) for the derivation of (16). Substituting the expression (13) for $\tilde{\hat{\rho}}_R(0)$ into Eq. (16) we obtain

$$\Delta S\left[\hat{\rho}_S(0)\right] = \frac{5}{9}\left[tr(\hat{\rho}_S^2(0)) - \frac{1}{N}\right]. \qquad (17)$$

Following the approach accepted in the thermodynamics of the informational processes (see [6]) we introduce the coefficient of efficiency of the information transfer η. By definition, $\eta \equiv \Delta I_R/\Delta S$. It follows from the law of the increase of entropy that $\eta \le 1$ for any evolution of the composite system. Comparing the expressions (14) and (17) we find that $\eta_0 = 3/5$ and it does not depend on the initial state of the sender in the optimal regime of the information exchange.

Let us now discuss the question on the relation between the energy transfer and the information transfer under the measurement of $\hat{O}_C$ in the model considered. For this purpose we determine the evolution of the average energies of the subsystems R and S: $E_R(t) \equiv tr(\hat{\rho}_R(t)\hat{H}_R)$ and $E_S(t) \equiv tr(\hat{\rho}_S(t)\hat{H}_S)$. Using Eqs. (7), (8), we obtain the simple equations of evolution of $E_R(t)$ and $E_S(t)$:

$$\frac{dE_R}{dt} = E_S - E_R, \qquad (18)$$

$$\frac{dE_S}{dt} = E_R - E_S. \qquad (19)$$

Note that the unitary equivalence of the Hamiltonians H_R and H_S was used for the derivation of (18), (19). It follows from these equations that the energy of the composite system conserves and the energy is transferred from the "hotter" to "colder" part until their energies becomes equal each other at the end of the measurement. It is clear that if the initial energies of the parts are equal each other, there is no transfer of energy under the measurement. We call such a regime the isoenergetic one. Let us compute the main characteristics of the information transfer process in the optimal isoenergetic regime. As the matter of fact, in this case, the formulation of the problem and the method of computing of the quantities ΔI_R, ΔS and η remains the same. For instance, to compute ΔI_R^e one should find the maximum of the functional (12) with respect to $\hat{\rho}_R(0)$ under two additional conditions: $tr(\hat{\rho}_R(0)) = 1$ and $tr(\hat{\rho}_R(0)\hat{H}_R) = tr(\hat{\rho}_S(0)\hat{H}_S)$. Using the Lagrange multipliers method we obtain, after simple computations, the optimal initial state of the part R

$$\hat{\rho}_R^e(0) = \frac{\hat{U}^+\hat{\rho}_S(0)\hat{U}}{3} + \frac{2}{3N}\hat{1}_R + \frac{2}{3}\frac{tr(\hat{\rho}_S(0)\hat{H}_S)}{tr(\hat{H}_R^2)}\hat{H}_R. \qquad (20)$$

Substituting this expression into equation (12) we obtain the maximum amount of information transferred in the isoenergetic regime of the measurement of $\hat{O}_C$

$$\Delta I_R^e\left\{\hat{\rho}_S(0)\right\} = \frac{1}{3}\left\{tr(\hat{\rho}_S^2(0)) - \frac{1}{N} - \frac{\left(tr(\hat{\rho}_S(0)\hat{H}_S)\right)^2}{tr(\hat{H}_R^2)}\right\} \qquad (21)$$

Note that the derivation of (21) was done under the additional condition $tr(\hat{H}_R) = 0$ that fixes the reference point for the energy. This condition does not influence on the generality of the results obtained.

We also present the result for the increment of the entropy of the sender S in the isoenergetic regime of the measurement of $\hat{O}_C$:

$$\Delta S^e\left\{\hat{\rho}_S(0)\right\} = \frac{5}{9}\left[tr(\hat{\rho}_S^2(0)) - \frac{1}{N} - \frac{\left(tr(\hat{\rho}_S(0)\hat{H}_S)\right)^2}{tr(\hat{H}_R^2)}\right] \qquad (22)$$

Comparing the expressions (21) and (22), we find that the coefficient of efficiency of the information transfer in such a regime η is also equal to 3/5.

To illustrate the expression (21) we consider the optimal process of the measurement for the simplest composite system C of the dimension 4 ($\dim C = 4$, $\dim R = \dim S = 2$) without energy exchange between its parts. We use the representation where the Hamiltonian of the sender is diagonal: $\hat{H}_S = \Delta_0\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$ with $2\Delta_0$, the distance between the energy levels of the part S. Let at the starting time of the measurement the state of S is described by the density matrix $\hat{\rho}_S(0) = \begin{pmatrix}a & c\\ c^* & b\end{pmatrix}$. Then $tr(\hat{\rho}_S\hat{H}_S) = \Delta_0(a-b)$, $tr(\hat{H}_R^2) = tr(\hat{H}_S^2) = 2\Delta_0^2$ and under accounting the normalization condition $a+b=1$ the expression (21) for ΔI_R^e takes the form

$$\begin{aligned}\Delta I_R^e\left\{\hat{\rho}_S(0)\right\} = \frac{1}{3}\left[a^2 + b^2 + 2|c|^2 - \frac{1}{2} - \frac{(a-b)^2}{2}\right] \\ = \frac{2}{3}|c|^2.\end{aligned} \qquad (23)$$

One can see that the maximum amount of information transferred to one of the parts under the measurement of $\hat{O}_C$ in the isoenergetic regime is determined by the

non-diagonal elements of the density matrix $\hat{\rho}_S(0)$ in the representation where $\hat{H}_S$ is diagonal.

Thus, we summarize the results of the paper. On the example of the simple model of the measurement in the composite system we have demonstrated the possibility of the exchange of information between its parts. We compute the main characteristics of such a process for the optimal initial states of the subsystems. One should emphasize that, in itself, the possibility of the recoherence of one part of the composite system under the measurement is determined entirely by the type of the measured quantity and by the initial state of the system, and it does not depend on the simplified assumptions used in our consideration. The generalization of the results to the case of unitary non-equivalent Hamiltonians of the parts R and S and the study of the information exchange process between the subsystems of different dimensions is postponed for further publications. In conclusion, we discuss shortly the possibility of the observation of the effect predicted in this paper - the recoherence of the state of the subsystem under the continuous measurement. For the first time, the suggestion to use the procedure of continuous measurement of the energy of a two-level system for the monitoring of the quantum transition was put forward in Ref. [7]. The experimental scheme of realization of this idea was also described in that paper. The object of the measurement is a polarized atom with the transition excited by resonant pumping. The electron beam that scatters on the atom (interacting with its dipole momentum) is used as a meter. Measuring the scattering angle one can obtain the energy of the probed atom at any time.

Using the ideas of Ref. [7], we describe the simplest, from our point of view, scheme of the experiment in which the effect of recoherence under the measurement in a composite system can emerge. For this purpose, we consider two crossed beams of neutral particles (neutrons) with the spins $s_1 = s_2 = 1/2$ that propagate close to each other in the area that contains a massive magnetic atom with the spin S. We imply the neutrons of the beam 1 are in the mixed state described by the density matrix $\hat{\rho}_1$ and the neutrons of the beam 2 - in the state described by the density matrix $\hat{\rho}_2$. It is assumed that two beams are synchronized in such a way that in time when the neutron of one beam moves close to the magnetic atom there is also the neutron of beam 2 near this atom. Since there is the exchange interaction between neutrons of different beams and between neutrons and the magnetic atom, one can consider that in such an experiment the massive atom provides the continuous measurement of the observable $\hat{O} = \hat{\mathbf{s}}_1\hat{\mathbf{s}}_2$ of the two-particle system. Let us remind now that the operator $\hat{\mathbf{s}}_1\hat{\mathbf{s}}_2$ is connected with the operator of spin permutation $\hat{T}$ by the relation $\hat{T} = (1 + \hat{\mathbf{s}}_1\hat{\mathbf{s}}_2/4)/2$ (see [8]). Taking into account the foregoing statements of the paper we arrive to the conclusion that under proper choice of the states of the beams $\hat{\rho}_1$ and $\hat{\rho}_2$ such an experiment should demonstrate the effect of recoherence of the state of neutrons of one of the beams. Comparing the interference pattern of the recoherred beam with the interference pattern of the test beam (obtained for the same $\hat{\rho}_1$ and $\hat{\rho}_2$, but without the "measuring" magnetic atom) one can check all main conclusions and relations of this paper.

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APPENDIX A

Let us present the proof of the statement of the paper on the monotonical increase of (linear) entropy of OQS, the interaction of which with the environment is described in the framework of the Lindblad equation (1) by the operators $\hat{R}$ and $\hat{R}^+$ that obey the relation $[\hat{R}, \hat{R}^+] = 0$.

Using Eq. (1) and the expression for the linear entropy $S[\hat{\rho}] = 1 - tr(\hat{\rho}^2)$ one can write the expression for the rate of change of the entropy

$$\frac{dS}{dt} = -2tr(\hat{\rho}\dot{\hat{\rho}}) = 2\gamma tr(\hat{R}^+\hat{R}\hat{\rho}^2 - \hat{\rho}\hat{R}\hat{\rho}\hat{R}^+). \tag{A1}$$

As follows from (A1) the monotonic increase of the entropy ($dS/dt \geq 0$) takes place if the following inequality is satisfied:

$$tr(\hat{R}^+\hat{R}\hat{\rho}^2) \geq tr(\hat{\rho}\hat{R}\hat{\rho}\hat{R}^+). \tag{A2}$$

To prove the inequality (A2) we use the Cauchy-Bunyakovsky-Schwarz (CBS) inequality

$$||\mathbf{A}||^2 \cdot ||\mathbf{B}||^2 \geq (\mathbf{AB})^2, \tag{A3}$$

that is valid in any linear space where the scalar product of two vectors $\mathbf{A}$ and $\mathbf{B}$ ($||\mathbf{A}|| \equiv \sqrt{\mathbf{AA}}$) is defined. We remind that for any pair of linear operators $\hat{A}$ and $\hat{B}$ acting in a vector space of finite dimension one can define the operation $\hat{A}\hat{B} \equiv tr(\hat{A}\hat{B}^+)$ that satisfies all axioms of the scalar product [9]. We choose the operator $\hat{\rho}\hat{R}$ as $\hat{A}$ and the operator $\hat{R}\hat{\rho}$ as $\hat{B}$ and write the CBS inequality (A3) for these operators:

$$tr(\hat{R}\hat{R}^+\rho^2)tr(\hat{R}^+\hat{R}\hat{\rho}^2) \geq [tr(\hat{\rho}\hat{R}\hat{\rho}\hat{R}^+)]^2. \tag{A4}$$

Here we use the possibility to do cyclic permutations of operators under the trace.

Using the commutativity of $\hat{R}$ and $\hat{R}^+$ and taking into account that $tr(\hat{R}^+\hat{R}\hat{\rho}^2) = tr(\hat{\rho}\hat{R}\hat{R}^+\hat{\rho}) \geq 0$ we obtain from (A4) the required inequality (A2). Thus, the statement on the monotonic increase of linear entropy in such OQS is proven.

APPENDIX B

In this appendix we consider two particular realizations (cases) of the alternative A1 (the decoherence of the

states of the parts under the continuous measurement in the composite system).

Case 1. The measurement of the additive variable.

Under such measurement the evolution of the state $\hat{\rho}_C(t)$ is described by the master equation of the CQM theory. This equation, written in the dimensionless form, reads as

$$\frac{d\hat{\rho}_C(t)}{dt} = -\frac{1}{2}\left[\hat{O}_C, \left[\hat{O}_C, \hat{\rho}_C\right]\right], \tag{B1}$$

where

$$\hat{O}_C = \hat{A}_R \otimes \hat{1}_S + \hat{1}_R \otimes \hat{B}_S. \tag{B2}$$

One can check directly that the general solution of Eq. (B1) has the form

$$\hat{\rho}_C(t) = \frac{1}{\sqrt{2\pi t}} \int_{-\infty}^{\infty} ds e^{-s^2/2t} e^{-i\hat{O}_C s} \hat{\rho}_C(0) e^{i\hat{O}_C s}. \tag{B3}$$

The factor $\exp(-i\hat{O}_C s)$ under the integral in (B3) can be presented as

$$\begin{aligned}\exp(-i\hat{O}_C s) &= \exp(-i(\hat{A}_R \otimes \hat{1}_S + \hat{1}_R \otimes \hat{B}_S)s) \\ &= (\exp(-i\hat{A}_R s) \otimes \hat{1}_S) \cdot (\hat{1}_R \otimes \exp(-i\hat{B}_S s)).\end{aligned} \tag{B4}$$

According to the quantum theory, $\hat{\rho}_R(t)$, the state of the part R, is determined by the relation

$$\rho^R_{ij}(t) = tr_S(\hat{\rho}_C(t)) \equiv \sum_\alpha \langle i\alpha|\hat{\rho}_C(t)|j\alpha\rangle, \tag{B5}$$

where $|i\alpha\rangle \equiv |i\rangle_R \otimes |\alpha\rangle_S$ is the orthogonal basis in C.

Using the expressions (B3), (B4) and the definition (B5) we find

$$\begin{aligned}\rho^R_{ij}(t) &= \frac{1}{\sqrt{2\pi t}} \int_{-\infty}^{\infty} ds e^{-\frac{s^2}{2t}} \sum_\alpha \langle i\alpha|e^{-i\hat{O}_C s}\hat{\rho}_C(0)e^{i\hat{O}_C s}|j\alpha\rangle \\ &= \sum_{kl} \frac{1}{\sqrt{2\pi t}} \int_{-\infty}^{\infty} ds e^{-s^2/2t} \langle i|e^{-i\hat{A}_R s}|k\rangle \\ &\quad \cdot \sum_\gamma \langle k\gamma|\hat{\rho}_C(0)|l\gamma\rangle\langle l|e^{i\hat{A}_R s}|j\rangle \\ &= \frac{1}{\sqrt{2\pi t}} \int_{-\infty}^{\infty} ds e^{-s^2/2t} \left(e^{-i\hat{A}_R s}\hat{\rho}_R(0)e^{i\hat{A}_R s}\right)_{ij}\end{aligned} \tag{B6}$$

Thus, one can see that the measurement of the additive quantity $\hat{O}_C$ is, in fact, reduced to the measurement of the quantities $\hat{A}_R$ and $\hat{B}_S$ in each of the subsystems and that is why it results in the decoherence of its states.

Case 2. The measurement of the multiplicative observable.

As above, the evolution of $\hat{\rho}_C(t)$ is determined by Eq. (B1), but the measured quantity $\hat{O}_C$ has the form

$$\hat{O}_C = \hat{A}_R \otimes \hat{B}_S. \tag{B7}$$

One can use the general form (B3) of the solution of Eq. (B1) and write $\hat{\rho}_C(t)$ in the basis $|i\rangle_R \otimes |\alpha\rangle_S$, where $|i\rangle_R$ are the eigenvectors of $\hat{A}_R$, and $|\alpha\rangle_S$ are the eigenvectors of $\hat{B}_S$. After simple calculations we find that

$$\langle i\alpha|\hat{\rho}_C(t)|j\beta\rangle = \langle i\alpha|\hat{\rho}_C(0)|j\beta\rangle e^{-(A_i B_\alpha - A_j B_\beta)^2 t}, \tag{B8}$$

where A_i and B_α are the eigenvalues of the operators $\hat{A}_R$ and $\hat{B}_S$ in the states $|i\rangle_R$ and $|\alpha\rangle_S$, respectively.

Let us compute the rate of change of the entropy of the subsystem R

$$\begin{aligned}\frac{dS_R}{dt} &= -\frac{d}{dt}tr(\hat{\rho}^2_R(t)) = -2\sum_{ij} \rho^R_{ij}(t)\dot{\rho}^R_{ji}(t) \\ &= 2\sum_{\alpha\beta ij} e^{-(B^2_\alpha + B^2_\beta)(A_i - A_j)^2 t}(A_i - A_j)^2 B^2_\beta \\ &\qquad \times\langle i\alpha|\hat{\rho}_C(0)|j\alpha\rangle\langle j\beta|\hat{\rho}_C(0)|i\beta\rangle\end{aligned} \tag{B9}$$

If one assumes that at the starting time of the measurement of $\hat{O}_C$ the state of the parts R and S are not correlated ($\hat{\rho}_C(0) = \hat{\rho}_R(0) \otimes \hat{\rho}_S(0)$), the expression (B9) is reduced to

$$\begin{aligned}\frac{dS_R}{dt} &= 2\sum_{\alpha\beta ij} (A_i - A_j)^2 B^2_\beta e^{-(B^2_\alpha + B^2_\beta)(A_i - A_j)^2 t} \\ &\qquad \times|\rho^R_{ij}(0)|^2 \rho^S_{\alpha\alpha}(0)\rho^S_{\beta\beta}(0).\end{aligned} \tag{B10}$$

One can see directly from (B10) that $dS_R/dt \geq 0$. The relation $dS_S/dt \geq 0$ can be obtained by the same way. Thus, we have proven the statement of the paper on the decoherence of the state of the subsystem R and S under the measurement of the multiplicative observable $\hat{O}_C$ for the case of uncorrelated initial states of R and S.

[1] W. H. Zurek, Phys. Rev D **24**, 1516 (1981); Phys. Rev D **26**, 1862 (1982).

[2] M. B. Mensky, Zh. Exp. Teor. Fiz. **77**, 1326 (1979) [Sov. Phys. JETP **50**, 667 (1979)]; Phys. Rev D **20**, 384 (1979); Usp. Fiz. Nauk **168**, 1017 (1998).

[3] G. Lindblad, Commun. Math. Phys. **48**, 119, (1976).

[4] J. von Neumann, Mathematical foundations of quantum mechanics, Princeton, 1955.

[5] L. Brillouin, Science and information theory, N.-Y, 1956.

[6] R.P.Poplavskii, Thermodymamics of information processes. Moskva, Nauka, 1981 (in Russian).

[7] J. Andretsch, M.B.Mensky, Phys. Rev A **56**, 44 (1997).

[8] R. Feynman, R. Leighton, M.Sands. The Feynman lectures on physics. V.3, Massachusetts-London, 1963.

[9] M. Marcus, H. Minc. A Survey of matrix theory and matrix inequalities, Boston, 1964.